

Takeo Uramoto

Kyushu University, Institute of Mathematics for Industry,
Japan

Title: *On algebraic language theory, Galois theory, and class field theory*

Abstract: In this talk, I will introduce some of my recent works on algebraic language theory, and its connection to galois theory and classical class field theory. Algebraic language theory generally concerns classification of formal languages by means of algebra; Eilenberg theory in particular focuses on classification of regular languages by means of finite semigroup theory, where typical results, say Schutzenberger's pioneering result in 1965, states that a regular language L can be defined by first-order formula of Buchi's monadic second-order logic on finite words if and only if its syntactic monoid $M(L)$ contains only trivial subgroups; this connects combinatorial/logical describability of regular languages with purely semigroup-theoretic property of their syntactic monoids; and Eilenberg theory has developed similar results on the connection between regular languages and finite monoids. These results in Eilenberg theory are analogous to classical results in galois theory, where, say, Galois' theorem states that the solutions of a polynomial equation $f(x)=0$ can be written by four arithmetic operations and taking roots if and only if its galois group is soluble. This talk first introduces that this analogy can be in fact justified by a suitable category-theoretic duality theorem; and then that this connection between Eilenberg's theory and Galois' theory sheds new light on classical class field theory, in particular, the theory of complex multiplication. I may also discuss some future directions related to these results.